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Massless Limits of Massive Tensor Fields

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Abstract

In order to construct a massive tensor theory with a smooth massless limit, we apply two kinds of gauge-fixing procedures, Nakanishi's one and the BRS one, to two models of massive tensor field. The first is of the Fierz-Pauli (FP) type, which describes a pure massive tensor field; the other is of the additional-scalar-ghost (ASG) type, which includes a scalar ghost in addition to an ordinary tensor field. It is shown that Nakanishi's procedure can eliminate massless singularities in both two models, while the BRS procedure regularizes the ASG model only. The BRS-regularized ASG model is most promising in constructing a complete nonlinear theory.

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§1. Introduction

In order to obtain a satisfactory formulation of the infrared problem in quantum gravity, we re-examine smooth massless limits of massive tensor field theories.

Two models are studied in the present paper: the first is of the Fierz-Pauli (FP) type; the other is of the additional-scalar-ghost (ASG) type. The FP model has been adopted as a standard model of massive tensor field because it describes a pure massive tensor with five degrees of freedom. Two-point functions of this model, however, take form quite different from the corresponding ones in the massless case. On the other hand, although the ASG model includes a scalar ghost in addition to an ordinary tensor field, there are some similarities between two-point functions of the ASG model and those in the massless case. [1)]

Kimura [2)] investigated a massless tensor field in general covariant gauge, proposing a model with ASG-type mass term as a good candidate for massive theory with a smooth massless limit. Fronsdal and Heidenreich [3)] succeeded in regularizing the FP model. They subtracted every massless singularity from the whole set of two-point functions by introducing two kinds of auxiliary fields of spin-1 and 0. Because the Lagrangian obtained is not so simple, however, it seems unsuitable for constructing a complete nonlinear theory.

In the present paper we apply to the two models two kinds of gauge-fixing procedures: Nakanishi's procedure and the BRS one. Nakanishi showed that simple addition of a gauge-fixing term to the free Lagrangian for an Abelian massive vector field can regularize massless singularities in the original theory. [4)] Applying this procedure to the case of tensor field, we find that both of the two models become free from massless singularities. The BRS gauge-fixing procedure has been recognized to have wide applicability. [5)] It is shown that massless singularities in the ASG model are in fact regularized by this procedure. For the FP model, however, this procedure does not work: there still remain massless singularities, though weaker than before the application of this procedure.

However, this is the story of a linearized world. Our main, not present but future, purpose is to construct a complete nonlinear theory of massive tensor field. For this purpose it is desirable to have linearized theories with higher symmetry properties. From this point of view the BRS procedure is more suitable than Nakanishi's. This is because the former installs BRS symmetry in the theory, while the latter does not implement any symmetry property. That means the BRS-regularized ASG model seems most promising for our purpose.

In §2, we review the case of Abelian vector field. This is to see how Nakanishi's and the BRS procedures work for regularizing massless singularities in the original massive theory. In §3, two models are presented for massive tensor field. Nakanishi's gauge-fixing procedure is applied to them in §4, while the BRS procedure applied in §5. Section 6 is devoted to

summary and discussion.

§2. Massive Vector Fields

2.1. *Massless vector*

We begin with a free Abelian massless vector field. The Lagrangian with the usual gauge-fixing term is *)

$$L = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + b\left(\partial^\mu A_\mu + \frac{\alpha}{2}b\right), \tag{2·1}$$

where b is the Nakanishi-Lautrup (NL) field and α is the gauge parameter. Field equations are

$$\Box A_\mu - (1-\alpha)\partial_\mu b = 0, \tag{2·2}$$

$$\partial^\mu A_\mu + \alpha b = 0, \tag{2·3}$$

$$\Box b = 0. \tag{2·4}$$

We also have

$$\Box \partial^\mu A_\mu = 0, \tag{2·5}$$

$$\Box^2 A_\mu = 0. \tag{2·6}$$

Two-point functions are

$$\langle A^\mu(x) A^\nu(y)\rangle = \frac{1}{\Box}\left[\eta^{\mu\nu} - (1-\alpha)\frac{\partial^\mu\partial^\nu}{\Box}\right]\delta(x-y), \tag{2·7}$$

$$\langle A^\mu(x) b(y)\rangle = \frac{\partial^\mu}{\Box}\delta(x-y), \tag{2·8}$$

$$\langle b(x) b(y)\rangle = 0. \tag{2·9}$$

2.2. *Massive vector*

In this case the Lagrangian is given by

$$L = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{m^2}{2}A_\mu A^\mu. \tag{2·10}$$

Field equations are

$$\left(\Box - m^2\right) A_\mu = 0, \tag{2·11}$$

$$\partial^\mu A_\mu = 0. \tag{2·12}$$

*) The metric used in the present paper is $\eta^{\mu\nu} = (-1, +1, +1, +1)$.

Two-point functions are

$$\langle A^\mu(x)A^\nu(y)\rangle = \frac{1}{\Box - m^2}\left[\eta^{\mu\nu} - \frac{\partial^\mu\partial^\nu}{m^2}\right]\delta(x-y), \tag{2·13}$$

which develop massless singularities in the limit of $m = 0$.

2.3. *Nakanishi's gauge-fixing procedure*

Following Nakanishi,[4] we add to the Lagrangian (2·10) the same gauge-fixing term as in the massless case:

$$L = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{m^2}{2}A_\mu A^\mu + b\left(\partial^\mu A_\mu + \frac{\alpha}{2}b\right). \tag{2·14}$$

This yields the following field equations:

$$\left(\Box - m^2\right)A_\mu - (1-\alpha)\partial_\mu b = 0, \tag{2·15}$$

$$\partial^\mu A_\mu + \alpha b = 0, \tag{2·16}$$

$$\left(\Box - \alpha m^2\right)b = 0, \tag{2·17}$$

and

$$\left(\Box - \alpha m^2\right)\partial^\mu A_\mu = 0, \tag{2·18}$$

$$\left(\Box - \alpha m^2\right)\left(\Box - m^2\right)A_\mu = 0. \tag{2·19}$$

Two-point functions in this case are

$$\langle A^\mu(x)A^\nu(y)\rangle = \frac{1}{\Box - m^2}\left[\eta^{\mu\nu} - (1-\alpha)\frac{\partial^\mu\partial^\nu}{\Box - \alpha m^2}\right]\delta(x-y), \tag{2·20}$$

$$\langle A^\mu(x)b(y)\rangle = \frac{\partial^\mu}{\Box - \alpha m^2}\delta(x-y), \tag{2·21}$$

$$\langle b(x)b(y)\rangle = -\frac{m^2}{\Box - \alpha m^2}\delta(x-y), \tag{2·22}$$

which show that the massless singularities in the original massive theory have been regularized by this procedure.

2.4. *BRS gauge-fixing procedure*

General consideration of this procedure has been developed by Izawa.[5] For the massive vector case, starting from the usual Lagrangian (2·10)

$$L_A = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{m^2}{2}A_\mu A^\mu, \tag{2·23}$$

we intend to carry out a field tranformation $A_\mu \to (A'_\mu, \theta)$ such that

$$A_\mu = A'_\mu - \frac{1}{m}\partial_\mu\theta, \tag{2·24}$$

$$\partial^\mu A'_\mu = 0. \tag{2·25}$$

Since the Lagrangian (2·23) is independent of the new variables (A'_μ, θ), it is invariant under the BRS transformation

$$\begin{cases} \delta A'_\mu = c_\mu, & \delta\bar{c}_\mu = ib_\mu, \\ \delta\theta = mc, & \delta\bar{c} = ib, \end{cases} \tag{2·26}$$

where the Faddeev-Popov (FP) ghosts (c_μ, c) and $(\bar{c}_\mu, \bar{c})$ as well as the NL fields (b_μ, b) have been introduced. To relate the old and new sets of variables, we add to the Lagrangian (2·23) the following BRS term:

$$\begin{aligned} L_{\mathrm{B}} =& -i\delta\left[\bar{c}^\mu\left(A_\mu - A'_\mu + \frac{1}{m}\partial_\mu\theta\right) + \bar{c}\left(\partial^\mu A'_\mu + \frac{\alpha}{2}b\right)\right] \\ =& \ b^\mu\left(A_\mu - A'_\mu + \frac{1}{m}\partial_\mu\theta\right) + b\left(\partial^\mu A'_\mu + \frac{\alpha}{2}b\right) \\ & - i\left(\bar{c}^\mu + \partial^\mu\bar{c}\right)\left(c_\mu - \partial_\mu c\right) + i\bar{c}\Box c. \end{aligned} \tag{2·27}$$

The path integral is given as

$$Z = \int \mathcal{D}A_\mu\mathcal{D}A'_\mu\mathcal{D}\theta\mathcal{D}b_\mu\mathcal{D}c_\mu\mathcal{D}\bar{c}_\mu\mathcal{D}b\mathcal{D}c\mathcal{D}\bar{c}\exp i\int d^4x\left[L_A + L_{\mathrm{B}}\right]. \tag{2·28}$$

Integrating over the variables $(b_\mu, A_\mu, c_\mu, \bar{c}_\mu)$ and overwriting A_μ on A'_μ, we obtain

$$Z = \int \mathcal{D}A_\mu\mathcal{D}\theta\mathcal{D}b\mathcal{D}c\mathcal{D}\bar{c}\exp i\int d^4x\, L_{\mathrm{T}}, \tag{2·29}$$

where

$$L_{\mathrm{T}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{m^2}{2}\left(A_\mu - \frac{1}{m}\partial_\mu\theta\right)^2 + b\left(\partial^\mu A_\mu + \frac{\alpha}{2}b\right) + i\bar{c}\Box c. \tag{2·30}$$

This Lagrangian is invariant under the following BRS transformation:

$$\delta A_\mu = \partial_\mu c, \qquad \delta\theta = mc, \qquad \delta\bar{c} = ib. \tag{2·31}$$

Since our model is Abelian, the FP ghosts $(c, \bar{c})$ decouple from any other field in the Lagrangian (2·30). When discussing field equations and two-point functions, therefore, we can neglect the last term in L_{T}. What we have obtained is nothing but the Stueckelberg Lagrangian. Field equations are

$$\left(\Box - m^2\right)A_\mu - (1-\alpha)\partial_\mu b + m\partial_\mu\theta = 0, \tag{2·32}$$

$$\partial^\mu A_\mu + \alpha b = 0, \tag{2·33}$$

$$\Box b = 0, \tag{2·34}$$

$$\Box\theta + \alpha m b = 0, \tag{2·35}$$

and

$$\Box\partial^\mu A_\mu = 0, \tag{2·36}$$

$$\Box^2\theta = 0, \tag{2·37}$$

$$\Box^2\left(\Box - m^2\right)A_\mu = 0. \tag{2·38}$$

Two-point functions are

$$\langle A^\mu(x)A^\nu(y)\rangle = \frac{1}{\Box - m^2}\left[\eta^{\mu\nu} - (1-\alpha)\frac{\partial^\mu\partial^\nu}{\Box} - \alpha m^2\frac{\partial^\mu\partial^\nu}{\Box^2}\right]\delta(x-y), \tag{2·39}$$

$$\langle A^\mu(x)b(y)\rangle = \frac{\partial^\mu}{\Box}\delta(x-y), \tag{2·40}$$

$$\langle A^\mu(x)\theta(y)\rangle = -\alpha m\frac{\partial^\mu}{\Box^2}\delta(x-y), \tag{2·41}$$

$$\langle b(x)b(y)\rangle = 0, \tag{2·42}$$

$$\langle b(x)\theta(y)\rangle = m\frac{1}{\Box}\delta(x-y), \tag{2·43}$$

$$\langle\theta(x)\theta(y)\rangle = \left(\frac{1}{\Box} - \alpha m^2\frac{1}{\Box^2}\right)\delta(x-y). \tag{2·44}$$

They are in fact singular-free in the massless limit. The field θ becomes redundant in this limit and the theory smoothly reduces to the usual massless theory.

§3. Massive Tensor Fields

3.1. *Massless tensor*

The Lagrangian with a gauge-fixing term is given by

$$\begin{aligned} L = & -\frac{1}{2}\left(\partial_\lambda h_{\mu\nu}\partial^\lambda h^{\mu\nu} - \partial_\lambda h\partial^\lambda h\right) + \partial_\lambda h_{\mu\nu}\partial^\nu h^{\mu\lambda} - \partial_\mu h^{\mu\nu}\partial_\nu h \\ & + b^\mu\left(\partial^\nu h_{\mu\nu} - \frac{1}{2}\partial_\mu h + \frac{\alpha}{2}b_\mu\right) \\ = & \ \frac{1}{2}h^{\mu\nu}\Lambda_{\mu\nu,\rho\sigma}h^{\rho\sigma} + b^\mu\left(\partial^\nu h_{\mu\nu} - \frac{1}{2}\partial_\mu h + \frac{\alpha}{2}b_\mu\right), \end{aligned} \tag{3·1}$$

where $h = h^\mu{}_\mu$ and

$$\begin{aligned} \Lambda_{\mu\nu,\rho\sigma} = & \left(\eta_{\mu\rho}\eta_{\nu\sigma} - \eta_{\mu\nu}\eta_{\rho\sigma}\right)\Box \\ & - \left(\eta_{\mu\rho}\partial_\nu\partial_\sigma + \eta_{\nu\sigma}\partial_\mu\partial_\rho\right) + \left(\eta_{\rho\sigma}\partial_\mu\partial_\nu + \eta_{\mu\nu}\partial_\rho\partial_\sigma\right). \end{aligned} \tag{3·2}$$

Field equations reduce to

$$\Box h_{\mu\nu} - \frac{1}{2}(1-2\alpha)\left(\partial_\mu b_\nu + \partial_\nu b_\mu\right) = 0, \tag{3·3}$$

$$\partial^\nu h_{\mu\nu} - \frac{1}{2}\partial_\mu h + \alpha b_\mu = 0, \tag{3·4}$$

$$\Box b_\mu = 0. \tag{3·5}$$

We also have

$$\Box\left(\partial^\nu h_{\mu\nu} - \frac{1}{2}\partial_\mu h\right) = 0, \tag{3·6}$$

$$\Box^2 h_{\mu\nu} = 0. \tag{3·7}$$

Two-point functions are calculated as [*)]

$$\begin{aligned}\langle h^{\mu\nu} h^{\rho\sigma}\rangle = \frac{1}{\Box}\Big\{&\frac{1}{2}\left(\eta^{\mu\rho}\eta^{\nu\sigma} + \eta^{\mu\sigma}\eta^{\nu\rho} - \eta^{\mu\nu}\eta^{\rho\sigma}\right)\\ &- \frac{1}{2}(1-2\alpha)\frac{1}{\Box}\left(\eta^{\mu\rho}\partial^\nu\partial^\sigma + \eta^{\mu\sigma}\partial^\nu\partial^\rho + \eta^{\nu\rho}\partial^\mu\partial^\sigma + \eta^{\nu\sigma}\partial^\mu\partial^\rho\right)\Big\}\,\delta,\end{aligned} \tag{3·8}$$

$$\langle h^{\mu\nu} b^\rho\rangle = \frac{1}{\Box}\left(\eta^{\mu\rho}\partial^\nu + \eta^{\nu\rho}\partial^\mu\right)\delta, \tag{3·9}$$

$$\langle b^\mu b^\rho\rangle = 0. \tag{3·10}$$

3.2. *Massive tensor of the FP type*

The FP-type Lagrangian for a massive tensor field is given by

$$L = \frac{1}{2}h^{\mu\nu}\Lambda_{\mu\nu,\rho\sigma}h^{\rho\sigma} - \frac{m^2}{2}\left(h^{\mu\nu}h_{\mu\nu} - h^2\right). \tag{3·11}$$

The set of field equations

$$\left(\Box - m^2\right)h_{\mu\nu} = 0, \tag{3·12}$$

$$\partial^\nu h_{\mu\nu} = 0, \tag{3·13}$$

$$h = 0 \tag{3·14}$$

shows that the Lagrangian (3·11) purely describes a massive tensor field with five degrees of freedom. This is the reason why this type of model has been taken as a standard one. However, some undesirable properties are owned by the two-point functions

$$\langle h^{\mu\nu} h^{\rho\sigma}\rangle = \frac{1}{\Box - m^2}\left\{\frac{1}{2}\left(\eta^{\mu\rho}\eta^{\nu\sigma} + \eta^{\mu\sigma}\eta^{\nu\rho} - \eta^{\mu\nu}\eta^{\rho\sigma}\right)\right.$$

[*)] Here and hereafter space-time coordinates are omitted in the field variables as well as in the δ-functions.

$$
-\frac{1}{2m^2}\left(\eta^{\mu\rho}\partial^\nu\partial^\sigma+\eta^{\mu\sigma}\partial^\nu\partial^\rho+\eta^{\nu\rho}\partial^\mu\partial^\sigma+\eta^{\nu\sigma}\partial^\mu\partial^\rho\right)
+\frac{2}{3}\left(\frac{1}{2}\eta^{\mu\nu}+\frac{\partial^\mu\partial^\nu}{m^2}\right)\left(\frac{1}{2}\eta^{\rho\sigma}+\frac{\partial^\rho\partial^\sigma}{m^2}\right)\Bigg\}\delta. \tag{3·15}
$$

The first and second terms on the right hand side of this expression have their own correspondents in the expressin (3·8). The massless singularities in the second term are the same as encountered in the case of vector field. The third term, however, develops higher massless singularities than the second term. Moreover, that term does not find its own correspondent in the expression (3·8). These points make this model difficult to regularize.

3.3. *Massive tensor of the ASG type*

We adopt a mass term slightly different from the FP-type one:

$$
L=\frac{1}{2}h^{\mu\nu}\varLambda_{\mu\nu,\rho\sigma}h^{\rho\sigma}-\frac{m^2}{2}\left(h^{\mu\nu}h_{\mu\nu}-\frac{1}{2}h^2\right). \tag{3·16}
$$

In this case, a field equation corressponding to (3·14) does not hold. We have only

$$
\left(\Box-m^2\right)h_{\mu\nu}=0, \tag{3·17}
$$

$$
\partial^\nu h_{\mu\nu}-\frac{1}{2}\partial_\mu h=0. \tag{3·18}
$$

Therefore, this model describes not only an ordinary tensor field but also an auxiliary scalar field. Two-point functions in this model are given as

$$
\begin{aligned}
\langle h^{\mu\nu}h^{\rho\sigma}\rangle &= \frac{1}{\Box-m^2}\Bigg\{\frac{1}{2}\left(\eta^{\mu\rho}\eta^{\nu\sigma}+\eta^{\mu\sigma}\eta^{\nu\rho}-\eta^{\mu\nu}\eta^{\rho\sigma}\right) \\
&\qquad -\frac{1}{2m^2}\left(\eta^{\mu\rho}\partial^\nu\partial^\sigma+\eta^{\mu\sigma}\partial^\nu\partial^\rho+\eta^{\nu\rho}\partial^\mu\partial^\sigma+\eta^{\nu\sigma}\partial^\mu\partial^\rho\right)\Bigg\}\delta
\end{aligned} \tag{3·19a}
$$

$$
\begin{aligned}
&= \frac{1}{\Box-m^2}\Bigg\{\frac{1}{2}\left(\eta^{\mu\rho}\eta^{\nu\sigma}+\eta^{\mu\sigma}\eta^{\nu\rho}-\eta^{\mu\nu}\eta^{\rho\sigma}\right) \\
&\qquad -\frac{1}{2m^2}\left(\eta^{\mu\rho}\partial^\nu\partial^\sigma+\eta^{\mu\sigma}\partial^\nu\partial^\rho+\eta^{\nu\rho}\partial^\mu\partial^\sigma+\eta^{\nu\sigma}\partial^\mu\partial^\rho\right) \\
&\qquad +\frac{2}{3}\left(\frac{1}{2}\eta^{\mu\nu}+\frac{\partial^\mu\partial^\nu}{m^2}\right)\left(\frac{1}{2}\eta^{\rho\sigma}+\frac{\partial^\rho\partial^\sigma}{m^2}\right)\Bigg\}\delta \\
&\quad -\frac{1}{\Box-m^2}\frac{2}{3}\left(\frac{1}{2}\eta^{\mu\nu}+\frac{\partial^\mu\partial^\nu}{m^2}\right)\left(\frac{1}{2}\eta^{\rho\sigma}+\frac{\partial^\rho\partial^\sigma}{m^2}\right)\delta.
\end{aligned} \tag{3·19b}
$$

Contrary to the FP model, the expression (3·19a) does not have a term like the third one on the right hand side of Eq.(3·15). This fact simplifies the procedures for constructing massless-regular theories. The expression (3·19b), however, shows that this model includes an additional scalar field with negative metric as well as an ordinary tensor field.

§4. Nakanishi's Gauge-Fixing Procedure

4.1. *FP model*

We supplement the FP Lagrangian (3·11) by the same gauge-fixing term as in the massless case:

$$L = \frac{1}{2} h^{\mu\nu} \Lambda_{\mu\nu,\rho\sigma} h^{\rho\sigma} - \frac{m^2}{2}\left(h^{\mu\nu}h_{\mu\nu} - h^2\right) + b^\mu \left(\partial^\nu h_{\mu\nu} - \frac{1}{2}\partial_\mu h + \frac{\alpha}{2} b_\mu\right). \tag{4·1}$$

Field equations obtained are

$$\left(\Box - m^2\right) h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu} m^2 h - \frac{1}{2}(1-2\alpha)\left(\partial_\mu b_\nu + \partial_\nu b_\mu\right) = 0, \tag{4·2}$$

$$\partial^\nu h_{\mu\nu} - \frac{1}{2}\partial_\mu h + \alpha b_\mu = 0, \tag{4·3}$$

$$- m^2 \partial_\mu h + \left(\Box - 2\alpha m^2\right) b_\mu = 0, \tag{4·4}$$

$$\left(\Box^2 - 4m^2\Box + 6\alpha m^4\right) h = 0. \tag{4·5}$$

We also have

$$\left(\Box - 2\alpha m^2\right)\left(\Box^2 - 4m^2\Box + 6\alpha m^4\right) b_\mu = 0, \tag{4·6}$$

$$\left(\Box - 2\alpha m^2\right)\left(\Box^2 - 4m^2\Box + 6\alpha m^4\right)\left(\partial^\nu h_{\mu\nu} - \frac{1}{2}\partial_\mu h\right) = 0, \tag{4·7}$$

$$\left(\Box - m^2\right)\left(\Box - 2\alpha m^2\right)\left(\Box^2 - 4m^2\Box + 6\alpha m^4\right) h_{\mu\nu} = 0. \tag{4·8}$$

Two-point functions in this case are

$$\begin{aligned}
\langle h^{\mu\nu} h^{\rho\sigma}\rangle = {} & \frac{1}{\Box - m^2}\left\{\frac{1}{2}\left(\eta^{\mu\rho}\eta^{\nu\sigma} + \eta^{\mu\sigma}\eta^{\nu\rho} - \eta^{\mu\nu}\eta^{\rho\sigma}\right)\right. \\
& - \frac{1}{2}(1-2\alpha)\frac{1}{\Box - 2\alpha m^2}\left(\eta^{\mu\rho}\partial^\nu\partial^\sigma + \eta^{\mu\sigma}\partial^\nu\partial^\rho + \eta^{\nu\rho}\partial^\mu\partial^\sigma + \eta^{\nu\sigma}\partial^\mu\partial^\rho\right) \\
& - \frac{m^2}{\Box^2 - 4m^2\Box + 6\alpha m^4}\left[\frac{1}{2}\left(\Box - 2\alpha m^2\right)\eta^{\mu\nu}\eta^{\rho\sigma}\right. \\
& \qquad + (1-2\alpha)\left(\eta^{\mu\nu}\partial^\rho\partial^\sigma + \eta^{\rho\sigma}\partial^\mu\partial^\nu\right) \\
& \qquad \left.\left. + 2(1-2\alpha)^2\frac{1}{\Box - 2\alpha m^2}\partial^\mu\partial^\nu\partial^\rho\partial^\sigma\right]\right\}\delta,
\end{aligned} \tag{4·9}$$

$$\begin{aligned}
\langle h^{\mu\nu} b^\rho\rangle = {} & \frac{1}{\Box - 2\alpha m^2}\left(\eta^{\mu\rho}\partial^\nu + \eta^{\nu\rho}\partial^\mu\right)\delta \\
& + \frac{m^2}{\Box^2 - 4m^2\Box + 6\alpha m^4}\left[\eta^{\mu\nu}\partial^\rho + 2(1-2\alpha)\frac{1}{\Box - 2\alpha m^2}\partial^\mu\partial^\nu\partial^\rho\right]\delta,
\end{aligned} \tag{4·10}$$

$$\langle b^\mu b^\rho\rangle = -\frac{2m^2}{\Box - 2\alpha m^2}\left[\eta^{\mu\rho} - \frac{\Box - m^2}{\Box^2 - 4m^2\Box + 6\alpha m^4}\partial^\mu\partial^\rho\right]\delta. \tag{4·11}$$

Although these expressins are complicated, smooth massless limits are seen to be assured for an arbitrary value of the gauge parameter α. For some special values of α, for example $\alpha = \frac{1}{2}$, we can have much simpler expressions.

4.2. *ASG model*

In this case the Lagrangian we take is

$$L = \frac{1}{2} h^{\mu\nu} \Lambda_{\mu\nu,\rho\sigma} h^{\rho\sigma} - \frac{m^2}{2} \left(h^{\mu\nu} h_{\mu\nu} - \frac{1}{2} h^2 \right) + b^\mu \left(\partial^\nu h_{\mu\nu} - \frac{1}{2} \partial_\mu h + \frac{\alpha}{2} b_\mu \right). \tag{4·12}$$

Field equations obtained here are much simpler than in the previous case:

$$\left(\Box - m^2 \right) h_{\mu\nu} - \frac{1}{2} (1 - 2\alpha) \left(\partial_\mu b_\nu + \partial_\nu b_\mu \right) = 0, \tag{4·13}$$

$$\partial^\nu h_{\mu\nu} - \frac{1}{2} \partial_\mu h + \alpha b_\mu = 0, \tag{4·14}$$

$$\left(\Box - 2\alpha m^2 \right) b_\mu = 0, \tag{4·15}$$

and further

$$\left(\Box - 2\alpha m^2 \right) \left(\partial^\nu h_{\mu\nu} - \frac{1}{2} \partial_\mu h \right) = 0, \tag{4·16}$$

$$\left(\Box - m^2 \right) \left(\Box - 2\alpha m^2 \right) h_{\mu\nu} = 0. \tag{4·17}$$

The structure of two-point functions also becomes simpler as follows:

$$\begin{aligned} \langle h^{\mu\nu} h^{\rho\sigma} \rangle = &\frac{1}{\Box - m^2} \left\{ \frac{1}{2} \left(\eta^{\mu\rho} \eta^{\nu\sigma} + \eta^{\mu\sigma} \eta^{\nu\rho} - \eta^{\mu\nu} \eta^{\rho\sigma} \right) \right. \\ &\left. - \frac{1}{2} (1 - 2\alpha) \frac{1}{\Box - 2\alpha m^2} \left(\eta^{\mu\rho} \partial^\nu \partial^\sigma + \eta^{\mu\sigma} \partial^\nu \partial^\rho + \eta^{\nu\rho} \partial^\mu \partial^\sigma + \eta^{\nu\sigma} \partial^\mu \partial^\rho \right) \right\} \delta, \end{aligned} \tag{4·18}$$

$$\langle h^{\mu\nu} b^\rho \rangle = \frac{1}{\Box - 2\alpha m^2} \left(\eta^{\mu\rho} \partial^\nu + \eta^{\nu\rho} \partial^\mu \right) \delta, \tag{4·19}$$

$$\langle b^\mu b^\rho \rangle = - \frac{2m^2}{\Box - 2\alpha m^2} \eta^{\mu\rho} \delta. \tag{4·20}$$

It is seen that Nakanishi's gauge-fixing procedure does work for regularizing massless singularities in the ASG model too.

§5. BRS Gauge-Fixing Procedure

5.1. *BRS procedure*

As in the vector case, we start from the usual massive-tensor Lagrangian without a gauge-fixing term:

$$L_h = \frac{1}{2} h^{\mu\nu} \Lambda_{\mu\nu,\rho\sigma} h^{\rho\sigma} - \frac{m^2}{2}\left(h^{\mu\nu}h_{\mu\nu} - ah^2\right), \tag{5·1}$$

where

$$a = \begin{cases} 1 & \text{for the FP model,} \\ \frac{1}{2} & \text{for the ASG model.} \end{cases} \tag{5·2}$$

We introduce a new set of variables $(h'_{\mu\nu}, \theta_\mu)$ to perform a field transformation $h_{\mu\nu} \to (h'_{\mu\nu}, \theta_\mu)$ such that

$$h_{\mu\nu} = h'_{\mu\nu} - \frac{1}{m}\left(\partial_\mu \theta_\nu + \partial_\nu \theta_\mu\right), \tag{5·3}$$

$$\partial^\nu h'_{\mu\nu} - \frac{1}{2}\partial_\mu h' = 0. \tag{5·4}$$

The Lagrangian (5·1), which is independent of the new variables, is invariant under the following BRS transformation:

$$\begin{cases} \delta h'_{\mu\nu} = c_{\mu\nu}, & \delta \bar{c}_{\mu\nu} = ib_{\mu\nu}, \\ \delta\theta_\mu = mc_\mu, & \delta\bar{c}_\mu = ib_\mu, \end{cases} \tag{5·5}$$

where $(c_{\mu\nu}, c_\mu)$ and $(\bar{c}_{\mu\nu}, \bar{c}_\mu)$ denote the FP ghosts and $(b_{\mu\nu}, b_\mu)$ indicate the NL fields. In order to perform the field transformation (5·3) with (5·4), we supplement the Lagrangian (5·1) by adding the following BRS gauge-fixing term:

$$\begin{aligned} L_{\mathrm{B}} =& -i\delta\left[\bar{c}^{\mu\nu}\left(h_{\mu\nu} - h'_{\mu\nu} + \frac{1}{m}\left(\partial_\mu\theta_\nu + \partial_\nu\theta_\mu\right)\right) + \bar{c}^\mu\left(\partial^\nu h'_{\mu\nu} - \frac{1}{2}\partial_\mu h' + \frac{\alpha}{2}b_\mu\right)\right] \\ =& \; b^{\mu\nu}\left(h_{\mu\nu} - h'_{\mu\nu} + \frac{1}{m}\left(\partial_\mu\theta_\nu + \partial_\nu\theta_\mu\right)\right) + b^\mu\left(\partial^\nu h'_{\mu\nu} - \frac{1}{2}\partial_\mu h' + \frac{\alpha}{2}b_\mu\right) \\ & - i\left(\bar{c}^{\mu\nu} + \frac{1}{2}\left(\partial^\mu\bar{c}^\nu + \partial^\nu\bar{c}^\mu - \eta^{\mu\nu}\partial_\rho\bar{c}^\rho\right)\right)\Big(c_{\mu\nu} - \left(\partial_\mu c_\nu + \partial_\nu c_\mu\right)\Big) + i\bar{c}^\mu\Box c_\mu. \end{aligned} \tag{5·6}$$

The path integral is given by

$$Z = \int \mathcal{D}h_{\mu\nu}\mathcal{D}h'_{\mu\nu}\mathcal{D}\theta_\mu\mathcal{D}b_{\mu\nu}\mathcal{D}c_{\mu\nu}\mathcal{D}\bar{c}_{\mu\nu}\mathcal{D}b_\mu\mathcal{D}c_\mu\mathcal{D}\bar{c}_\mu \exp i\int d^4x\left[L_h + L_{\mathrm{B}}\right]. \tag{5·7}$$

We integrate out with respect to the variables $(b_{\mu\nu}, h_{\mu\nu}, c_{\mu\nu}, \bar{c}_{\mu\nu})$, and then write $h_{\mu\nu}$ over $h'_{\mu\nu}$. The result is

$$Z = \int \mathcal{D}h_{\mu\nu}\mathcal{D}\theta_\mu\mathcal{D}b_\mu\mathcal{D}c_\mu\mathcal{D}\bar{c}_\mu \exp i\int d^4x\, L_{\mathrm{T}}, \tag{5·8}$$

where

$$L_{\mathrm{T}} = \frac{1}{2}h^{\mu\nu}\Lambda_{\mu\nu,\rho\sigma}h^{\rho\sigma}$$
$$- \frac{m^2}{2}\left[\left(h_{\mu\nu} - \frac{1}{m}\left(\partial_\mu\theta_\nu + \partial_\nu\theta_\mu\right)\right)^2 - a\left(h - \frac{2}{m}\partial^\mu\theta_\mu\right)^2\right]$$
$$+ b^\mu\left(\partial^\nu h_{\mu\nu} - \frac{1}{2}\partial_\mu h + \frac{\alpha}{2}b_\mu\right) + i\bar{c}^\mu\Box c_\mu. \tag{5·9}$$

This Lagrangian is invariant under the following BRS transformation:

$$\delta h_{\mu\nu} = \partial_\mu c_\nu + \partial_\nu c_\mu, \qquad \delta\theta_\mu = mc_\mu, \qquad \delta\bar{c}_\mu = ib_\mu. \tag{5·10}$$

For an Abelian case, which is the case we consider, we can neglect the last term in the Lagrangian (5·9) because the FP ghosts decouple from the other fields.

5.2. *FP model*

We put $a = 1$ and omit the FP-ghost term in the Lagrangian (5·9) to have

$$L = \frac{1}{2}h^{\mu\nu}\Lambda_{\mu\nu,\rho\sigma}h^{\rho\sigma}$$
$$- \frac{m^2}{2}\left(h^{\mu\nu}h_{\mu\nu} - h^2\right) - 2m\theta^\mu\left(\partial^\nu h_{\mu\nu} - \partial_\mu h\right) - \frac{1}{2}\left(\partial_\mu\theta_\nu - \partial_\nu\theta_\mu\right)^2$$
$$+ b^\mu\left(\partial^\nu h_{\mu\nu} - \frac{1}{2}\partial_\mu h + \frac{\alpha}{2}b_\mu\right). \tag{5·11}$$

Field equations derived from this Lagrangian are

$$\left(\Box - m^2\right)h_{\mu\nu} + m\left(\partial_\mu\theta_\nu + \partial_\nu\theta_\mu\right)$$
$$- \frac{1}{2}(1 - 2\alpha)\left(\partial_\mu b_\nu + \partial_\nu b_\mu\right) + \frac{1}{6}\eta_{\mu\nu}\partial^\rho b_\rho = 0, \tag{5·12}$$

$$\partial^\nu h_{\mu\nu} - \frac{1}{2}\partial_\mu h + \alpha b_\mu = 0, \tag{5·13}$$

$$\Box h + 2\alpha\partial^\mu b_\mu = 0, \tag{5·14}$$

$$\Box\theta_\mu + \alpha m b_\mu - \frac{1}{6m}\partial_\mu\partial^\nu b_\nu = 0, \tag{5·15}$$

$$\Box b_\mu = 0. \tag{5·16}$$

We also have

$$\Box\left(\partial^\nu h_{\mu\nu} - \frac{1}{2}\partial_\mu h\right) = 0, \tag{5·17}$$

$$\Box^2 h = 0, \tag{5·18}$$

$$\Box^2\theta_\mu = 0, \tag{5·19}$$

$$\Box^2\left(\Box - m^2\right)h_{\mu\nu} = 0. \tag{5·20}$$

Two-point functions obtained are the following:

$$
\begin{aligned}
\langle h^{\mu\nu} h^{\rho\sigma} \rangle &= \frac{1}{\Box - m^2} \left\{ \frac{1}{2} \left(\eta^{\mu\rho}\eta^{\nu\sigma} + \eta^{\mu\sigma}\eta^{\nu\rho} - \eta^{\mu\nu}\eta^{\rho\sigma} \right) \right. \\
&\quad - \frac{1}{2} \left[(1-2\alpha)\frac{1}{\Box} + 2\alpha \frac{m^2}{\Box^2} \right] \left(\eta^{\mu\rho}\partial^\nu\partial^\sigma + \eta^{\mu\sigma}\partial^\nu\partial^\rho + \eta^{\nu\rho}\partial^\mu\partial^\sigma + \eta^{\nu\sigma}\partial^\mu\partial^\rho \right) \\
&\quad \left. + \frac{2}{3} \left(\frac{1}{2}\eta^{\mu\nu} + \frac{\partial^\mu\partial^\nu}{\Box} \right) \left(\frac{1}{2}\eta^{\rho\sigma} + \frac{\partial^\rho\partial^\sigma}{\Box} \right) \right\} \delta, \qquad (5{\cdot}21) \\
\langle h^{\mu\nu} b^\rho \rangle &= \frac{1}{\Box} \left(\eta^{\mu\rho}\partial^\nu + \eta^{\nu\rho}\partial^\mu \right) \delta, \qquad (5{\cdot}22) \\
\langle h^{\mu\nu} \theta^\rho \rangle &= \left\{ \frac{1}{6m}\frac{1}{\Box}\eta^{\mu\nu}\partial^\rho - \alpha m \frac{1}{\Box^2} \left(\eta^{\mu\rho}\partial^\nu + \eta^{\nu\rho}\partial^\mu \right) + \frac{1}{3m}\frac{1}{\Box^2}\partial^\mu\partial^\nu\partial^\rho \right\} \delta, \qquad (5{\cdot}23) \\
\langle b^\mu b^\rho \rangle &= 0, \qquad (5{\cdot}24) \\
\langle b^\mu \theta^\rho \rangle &= \frac{m}{\Box}\eta^{\mu\rho}\delta, \qquad (5{\cdot}25) \\
\langle \theta^\mu \theta^\rho \rangle &= \left\{ \frac{1}{2}\frac{1}{\Box} \left(1 - 2\alpha \frac{m^2}{\Box} \right) \eta^{\mu\rho} - \frac{1}{6m^2}\frac{1}{\Box} \left(1 - \frac{m^2}{\Box} \right) \partial^\mu\partial^\rho \right\} \delta. \qquad (5{\cdot}26)
\end{aligned}
$$

We see there still remain massless singularities. The singularities found in (3·15) have been driven away indeed. However, the new singularities, though weaker than the original ones, have appeared in the θ-sector (5·23) and (5·26). It follows that the BRS gauge-fixing procedure cannot drive away all the massless singularities of the FP model although this procedure does reduce the degree of singularities.

5.3. *ASG model*

In this case we set $a = \frac{1}{2}$ in the Lagrangian (5·9). Neglecting the FP-ghost term again, we have

$$
\begin{aligned}
L &= \frac{1}{2} h^{\mu\nu} \Lambda_{\mu\nu,\rho\sigma} h^{\rho\sigma} \\
&\quad - \frac{m^2}{2} \left(h^{\mu\nu}h_{\mu\nu} - \frac{1}{2}h^2 \right) - 2m\theta^\mu \left(\partial^\nu h_{\mu\nu} - \frac{1}{2}\partial_\mu h \right) - \partial_\mu\theta_\nu\partial^\mu\theta^\nu \\
&\quad + b^\mu \left(\partial^\nu h_{\mu\nu} - \frac{1}{2}\partial_\mu h + \frac{\alpha}{2} b_\mu \right). \qquad (5{\cdot}27)
\end{aligned}
$$

Field equations in this case are

$$
\begin{aligned}
\left(\Box - m^2 \right) h_{\mu\nu} + m \left(\partial_\mu\theta_\nu + \partial_\nu\theta_\mu \right) & \\
- \frac{1}{2}(1-2\alpha) \left(\partial_\mu b_\nu + \partial_\nu b_\mu \right) &= 0, \qquad (5{\cdot}28) \\
\partial^\nu h_{\mu\nu} - \frac{1}{2}\partial_\mu h + \alpha b_\mu &= 0, \qquad (5{\cdot}29) \\
\Box\theta_\mu + \alpha m b_\mu &= 0, \qquad (5{\cdot}30) \\
\Box b_\mu &= 0. \qquad (5{\cdot}31)
\end{aligned}
$$

Further we have

$$\Box\left(\partial^\nu h_{\mu\nu} - \frac{1}{2}\partial_\mu h\right) = 0, \tag{5·32}$$

$$\Box^2\theta_\mu = 0, \tag{5·33}$$

$$\Box^2\left(\Box - m^2\right) h_{\mu\nu} = 0. \tag{5·34}$$

Two-point functions are calculated as

$$\begin{aligned}\langle h^{\mu\nu} h^{\rho\sigma}\rangle = &\frac{1}{\Box - m^2}\left\{\frac{1}{2}\left(\eta^{\mu\rho}\eta^{\nu\sigma} + \eta^{\mu\sigma}\eta^{\nu\rho} - \eta^{\mu\nu}\eta^{\rho\sigma}\right)\right.\\ &\left. - \frac{1}{2}\left[(1-2\alpha)\frac{1}{\Box} + 2\alpha\frac{m^2}{\Box^2}\right]\left(\eta^{\mu\rho}\partial^\nu\partial^\sigma + \eta^{\mu\sigma}\partial^\nu\partial^\rho + \eta^{\nu\rho}\partial^\mu\partial^\sigma + \eta^{\nu\sigma}\partial^\mu\partial^\rho\right)\right\}\delta,\end{aligned} \tag{5·35}$$

$$\langle h^{\mu\nu} b^\rho\rangle = \frac{1}{\Box}\left(\eta^{\mu\rho}\partial^\nu + \eta^{\nu\rho}\partial^\mu\right)\delta, \tag{5·36}$$

$$\langle h^{\mu\nu}\theta^\rho\rangle = -\alpha m\frac{1}{\Box^2}\left(\eta^{\mu\rho}\partial^\nu + \eta^{\nu\rho}\partial^\mu\right)\delta, \tag{5·37}$$

$$\langle b^\mu b^\rho\rangle = 0, \tag{5·38}$$

$$\langle b^\mu \theta^\rho\rangle = \frac{m}{\Box}\eta^{\mu\rho}\delta, \tag{5·39}$$

$$\langle \theta^\mu \theta^\rho\rangle = \frac{1}{2}\frac{1}{\Box}\left(1 - 2\alpha\frac{m^2}{\Box}\right)\eta^{\mu\rho}\delta. \tag{5·40}$$

Compare these expressions with the corresponding ones (3·19a) for the original ASG model. It is seen that the BRS gauge-fixing procedure have been able to regularize the massless singularities involved in the ASG model.

§6. Summary and Discussion

In this paper we have studied how to construct massive tensor theories with smooth massless limits. We have taken up the FP and ASG models for a linearized massive tensor field, and applied Nakanishi's and the BRS gauge-fixing procedures to each model. It has been found that the ASG model can be regularized by both of the procedures, while the FP model only by Nakanishi's procedure. We have thus obtained three kinds of regularized massive tensor theories without massless singularities: N-regularized FP model, N-regularized ASG model and BRS-regularized ASG model.

In order to construct a complete nonlinear theory, it is desirable to have linearized theories with higher symmetry properties. BRS symmetry seems to play an essential role in this respect. The BRS procedure just provides this symmetry, but Nakanishi's procedure does not. It follows that the BRS-regularized ASG model may be most promising for our purpose. Detailed discussions along this line will be made in a future publication.

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